NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

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AN APPROXIMATE DETERMINATION OF THE LIFT OF SLENDER CYLINDRICAL BODIES AND WING-BODY COMBINATIONS AT VERY HIGH SUPERSONIC SPEEDS

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SUMMARY

A theory for very high supersonic flow is applied to determine a first approximation to the pressure distribution, lift, and drag due to lift for a slender cylindrical body of revolution and a wing-body combination. The method is applicable only if the altitude of flight is low enough for the gas dynamics type of flow to exist, if the angle of attack is large, and if the Mach number is extremely large. The theory used was derived on the basis of a ratio of specific heats equal to unity.

The method indicates that the flow effectively separates from the body before reaching the widest part of the cross section. The boundary of this region is determined. Certain load concentrations are found near the wing-body junction of the configuration investigated.

INTRODUCTION

Long-range rocket-propelled missiles sometimes reach very high angles of attack at high supersonic speeds as they re-enter the dense lower atmosphere. These high angles of attack may be reached accidentally because of the lack of control at higher altitudes or because of the lift required to level out the flight path and to zoom the rocket upward again.

The most efficient shape for obtaining lift at very high supersonic speeds is a body with a flat lower surface as pointed out in reference 1. The pressure distribution for such a body is the same as that of a flat plate and can be calculated by the method of reference 2. A slight extension of this method is necessary, however, before the lift of the more common type of missile (one with circular cross section) can be determined.

A method is presented in reference 2 for approximately predicting the drag of three-dimensional shapes and the lift and drag of two-dimensional shapes at very high supersonic speeds. An extension of the previous work to include the lift and drag of slender cylindrical bodies at high angles of attack is given herein. More specifically, the present paper is confined to a consideration of the forces on that part of a circular cylinder which is not influenced by the nose of the body and to a discussion of the spanwise load distribution of a wing-body combination at high angles of attack. Determination of the pressures on the nose and the effect of nose pressures on body pressures are left for future research.

SYMBOLS

C_D	drag coefficient due to lift based on plan area of cylinder
C_L	lift coefficient based on plan area of cylinder
C_N	normal-force coefficient based on plan area of cylinder
C_1, C_2, and C_3	constants in equation for the shape of the shock wave
D	drag
d	diameter
L	lift
l	length of section of body under consideration
M	momentum
P	pressure coefficient; any point
r	radius
V	free-stream velocity
x,y	coordinate axes
α	angle of attack
β	angle between surface, or shock wave, and free-stream direction
γ	ratio of specific heats

ρ	free-stream density

Subscripts:

1	point where high-density region separates from body
2	point where high-density region hits wing
h	horizontal
i	end point of integration
N	normal
n	based on normal velocity component
v	vertical

METHOD OF ANALYSIS

Figure 1 is a schematic representation of a missile, with the hatched area being that suitable for investigation by the present method.

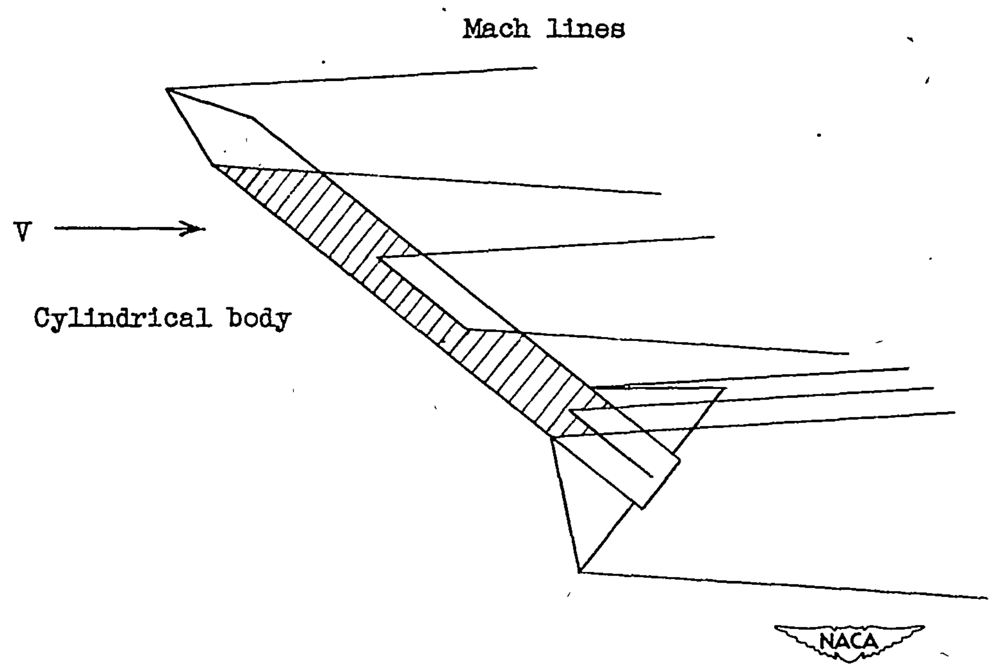

Figure 1.- Missile with region of application of present method (hatched area).

A method for determining the pressures over several aerodynamic shapes at very high supersonic speeds in the gas-dynamics flow regime is given in reference 2. The method is based on the fact that the pressure coefficient at any point on a body approaches a limiting value as the Mach number is increased. As the speed increases, the shock wave wraps more tightly about the body, and the region of disturbed flow around the body becomes relatively small. Reference 2 showed that the flow pattern and, hence, velocities, centrifugal forces, and surface pressures are not affected greatly by the value chosen for the ratio of specific heats γ. Since the selection of $\gamma = 1.0$ simplifies the problem considerably without modifying the results very much, this value of γ is used in the present analysis.

Figure 1 clearly shows the similarity between a cylinder at high angle of attack and a wing swept back outside the Mach cone. As on a sweptback wing, the pressures on the cylinder are determined only by the component of the free-stream velocity normal to the axis of the cylinder. The combination of free-stream Mach number and angle of attack must be such that the component V_N is much greater than the local speed of sound in order for the calculations to be valid. For a given forward Mach number, therefore, the equations are more accurate at high angles of attack. The problem has thus been reduced to the investigation of the pressures over a circular cylinder normal to the stream of air moving at V_N.

A schematic representation of the general flow pattern based on the results of reference 2 is given in figure 2. The thickness of the high-pressure region has been exaggerated for clarity. Also, the flow is shown to separate from the surface at some point which is to be calculated subsequently in this section.

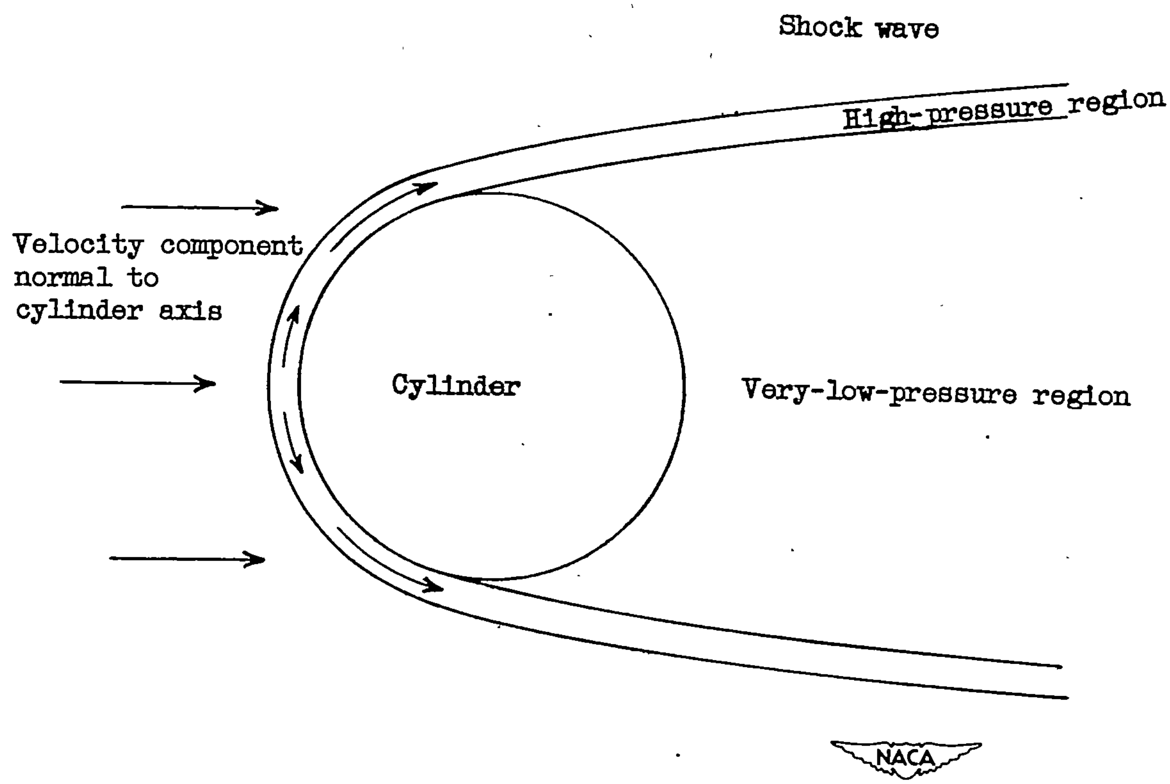

Figure 2.- Schematic flow pattern.

In figure 3 are shown the angle and points that are used in the calculations.

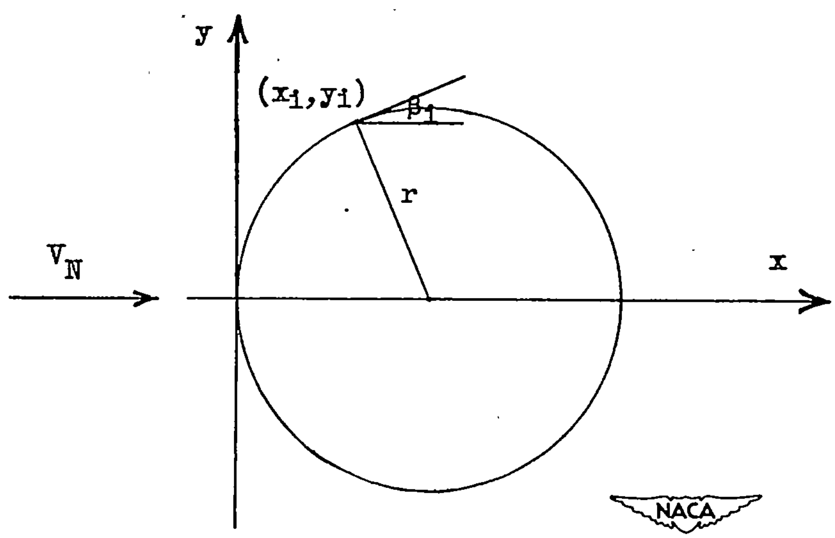

Figure 3.- Geometry of the analysis

The pressure coefficient P at any point (x_i,y_i) is given in reference 2 as

$$P_i = 2\ \sin^2\beta_i + 2\ \sin\ \beta_i\ \frac{d\beta_i}{dy_i}\int_0^{y_i} \cos\ \beta\ dy \qquad (1)$$

For the circular cylinder, $\frac{y}{r} = \cos\ \beta$ and $\frac{d\beta}{dy} = -\frac{1}{r\ \sin\ \beta}$. Then, the pressure coefficient for the circular cylinder at any point is

$$\begin{aligned} P_i &= 2\left(1 - \cos^2\beta_i\right) - \frac{2\ \sin\ \beta_i}{r\ \sin\ \beta_i}\int_0^{y_i} \cos\ \beta\ dy \\ &= 2\left(1 - \frac{{y_i}^2}{r^2}\right) - \frac{{y_i}^2}{r^2} \\ &= 2 - 3\frac{{y_i}^2}{r^2} \qquad (2) \end{aligned}$$

A sketch of the pressure distribution over the cylinder is given as figure 4; a more detailed plot of the pressure distribution is given in figure 5.

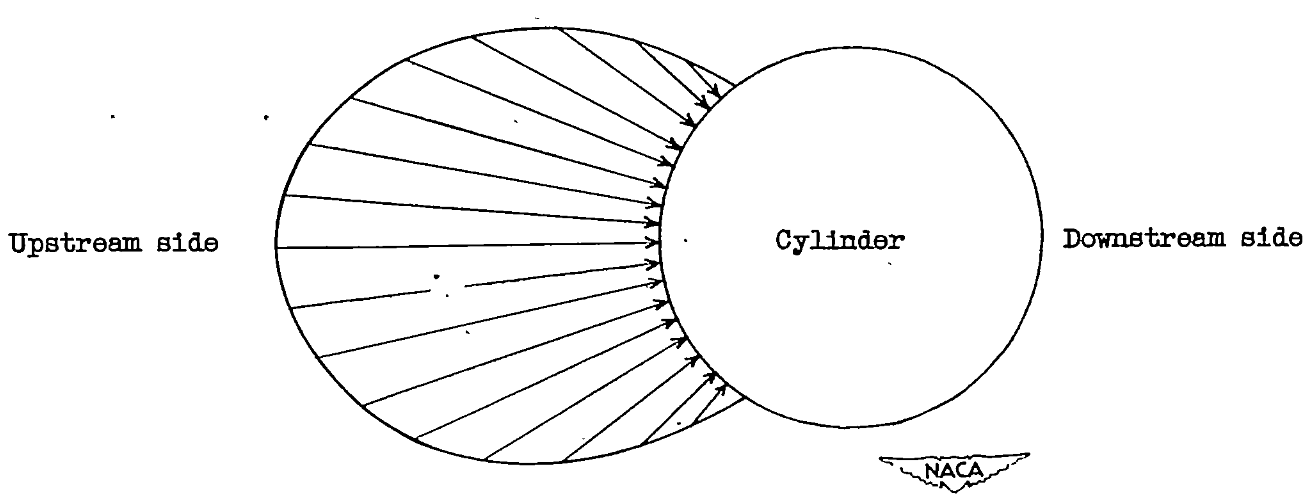

Figure 4.- Pressure distribution over the cylinder.

The limiting negative pressure coefficient at very high supersonic speeds is quite close to zero, and for all practical purposes negative pressure coefficients can be assumed to be unattainable at these speeds. The centrifugal force due to the curvature of the flow is equal to the difference in pressure between the shock wave and the surface of the cylinder. When the calculated surface pressure coefficient reaches zero, the shock wave must diverge from the body inasmuch as the pressure drop across the disturbed air layer is no longer sufficient to produce the flow curvature required for following the surface. The separation point (x_1,y_1) where the shock wave can no longer follow the surface curvature is found by setting the pressure coefficient equal to zero; that is,

$$P_1 = 2 - \frac{3{y_1}^2}{r^2} = 0$$

therefore,

$$\frac{y_1}{r} = \sqrt{\frac{2}{3}} = 0.8165 \tag{3}$$

Since for a body with a given diameter the lift and drag increase directly with the length of the body, the force coefficients are based on the plan area of the cylinder - that is, the product of length and diameter. The normal-force coefficient based on the dynamic pressure corresponding to the normal velocity component V_N is

$$\begin{aligned} C_{N_n} &= \frac{2l \int_0^{\sqrt{\frac{2}{3}}\,r} P_i \, dy}{ld} \\ &= \frac{2}{d} \int_0^{\sqrt{\frac{2}{3}}\,r} \left(2 - 3\frac{{y_i}^2}{r^2}\right) dy \\ &= 1.089 \end{aligned} \tag{4}$$

Based on free-stream dynamic pressure, the normal-force coefficient is

$$C_N = C_{N_n}\left(\frac{V_N}{V}\right)^2 = 1.089 \sin^2\alpha \tag{5}$$

Then, the lift coefficient is

$$C_L = 1.089 \sin^2\alpha \cos \alpha \tag{6}$$

and the drag coefficient due to lift is

$$C_D = 1.089 \sin^3\alpha \tag{7}$$

The lift-drag ratio, therefore, is

$$\frac{L}{D} = \cot \alpha \tag{8}$$

Equations (6), (7), and (8) are plotted in figures 6, 7, and 8, respectively. Maximum lift is obtained at an angle of attack of about 55° (fig. 6); the corresponding drag is very high (fig. 7). Because of the neglect of viscous drag, very high lift-drag ratios are indicated for low angles of attack (fig. 8). The method should not, however, be applied at low angles.

The shape of the shock wave that wraps about the cylindrical body can be estimated from momentum considerations. The "zero-pressure streamline method" of reference 2 assumes that the very-high-density region behind the shock wave is very narrow and is bounded on one side by shock pressure and on the other side by negligible pressure. Since the shock wave is one of the boundaries of the very-narrow, high-density region, the direction of the shock wave at any point is the same as the direction of the total momentum in the high-density region at that same point; therefore, the slope of the shock wave at any point (x_i,y_i) is

$$\left(\frac{dy}{dx}\right)_i = \left(\frac{M_v}{M_h}\right)_i \tag{9}$$

The vertical momentum at the point (x_1,y_1) where the flow effectively separates from the body is given by the following equation:

$$M_{v_1} = \rho V_N{}^2 \sin \beta_1 \int_0^{y_1} \cos \beta \, dy$$

The horizontal momentum at this same point is given as

$$M_{h_1} = \rho V_N^2 \cos\beta_1 \int_0^{y_1} \cos\beta \, dy$$

If no further loss in momentum is assumed to take place after the high-density flow separates from the body, the vertical momentum remains constant and equal to the value at the point (x_1,y_1). The horizontal momentum, however, at some point (x_i,y_i) beyond the point of separation is the sum of the horizontal momentum at the point (x_1,y_1) and the horizontal momentum added from the free stream between the points (x_1,y_1) and (x_i,y_i). Thus,

$$M_{h_i} = M_{h_1} + \rho V_N^2 \int_{y_1}^{y_i} dy$$

$$= \rho V_N^2\left(\cos\beta_1 \int_0^{y_1} \cos\beta \, dy - y_1 + y_i\right)$$

Substituting the foregoing relations for M_{v_i} and M_{h_i} in equation (9) and letting

$$C_1 = 2\left(\cos\beta_1 \int_0^{y_1} \cos\beta \, dy - y_1\right)$$

and

$$C_2 = -2 \sin\beta_1 \int_0^{y_1} \cos\beta \, dy$$

give

$$\frac{dy_1}{dx_1} = \frac{-C_2}{C_1 + 2y_1}$$

Integrating yields the following relation:

$$y_1^2 + C_1y_1 + C_2x_1 = C_3 \tag{10}$$

The constant C_3 may be obtained by evaluating the left side of the equation at the point (x_1,y_1). The equation of the shock wave is, thus, parabolic, and the constants involved vary with the shape of the body.

For the case considered herein (cylinder with $r = 1.0$) the constants may be evaluated from the following relations (see fig. 3):

$$\sin \beta = 1 - x$$

$$\cos \beta = y$$

$$\int_0^{y_1} \cos \beta \, dy = \frac{y_1^2}{2}$$

Since y_1 was found to be 0.8165 (equation (3)), then

$$x_1 = r - \sqrt{r^2 - y_1^2} = 0.4231$$

Evaluating the constants and substituting them in equation (10) yield the following equation for the shock wave after it leaves the cylinder:

$$y_1^2 - 1.089y_1 - 0.385x_1 + 0.385 = 0 \tag{11}$$

Equation (11) is plotted in figure 9.

SPANWISE LOAD DISTRIBUTION

The method used herein for obtaining the spanwise load distribution is limited to wing sections with flat lower surfaces. The spanwise load distribution can then be calculated in three parts.

Load distribution over cylindrical body.- The load distribution over the cylindrical body can be obtained from equation (2) or from figure 5.

Load distribution along wing outboard from point where high-density air hits wing.- Since a complete loss in momentum (in the direction normal to the body axis) occurs on the lower surfaces of the wings just as on the leading edge of the cylindrical body, the pressure coefficient is, therefore, the same at both places. This pressure coefficient is the peak pressure coefficient of the cylindrical body, and the value is 2.0.

Concentrated load in high-density flow that separates from cylinder.- The pressure near the wing-body intersection follows from elementary considerations. Since the normal-force coefficient (1.089) of the body based on the dynamic pressure for the velocity normal to the axis of the body is less than that due to a complete loss in momentum (2.0), appreciable momentum remains in the air stream. This momentum is concentrated into a very narrow spanwise distance (zero for the case of $\gamma = 1.0$ because the density goes to infinity) and, finally, is completely lost at the wing surface (near the wing root). The momentum of the flow normal to the axis of the body is all lost (part at the body and part near the wing-body juncture); therefore, the total lift for a unit length of wing and body is the same as that for a continuous wing. Inasmuch as the lift coefficient over the body is about one-half that over a wing, the lift coefficient at the wing root must then be higher than that of a wing alone; thus, very high peak pressures exist near the wing root. The theory indicates that for the limiting case of extremely high Mach numbers the pressure coefficient is infinity for an infinitesimal distance along the span (point P_2 in fig. 10). Point P_2 can be found from the equation of the shock wave (equation (11)). For finite Mach numbers the peak pressures are expected to be finite and the pressure distribution is expected to have less abrupt changes.

The part of the body ahead of or behind the section influencing the wing develops lift in the manner shown in figures 5 and 6.

CONCLUDING REMARKS

An approximate theory for very high supersonic speeds is applied to determine the lift and drag due to lift for a cylindrical body and a wing-body combination at high angles of attack in a gas dynamics type of flow. For a body with a given diameter the lift is directly proportional to the body length. The lift of the circular body is approximately one-half that of a wing of the same plan area. The wing roots have extremely high pressures because they convert the flow around the local body sections into additional lift. The wing and the corresponding part of the body develop a combined lift equal to that of a continuous wing.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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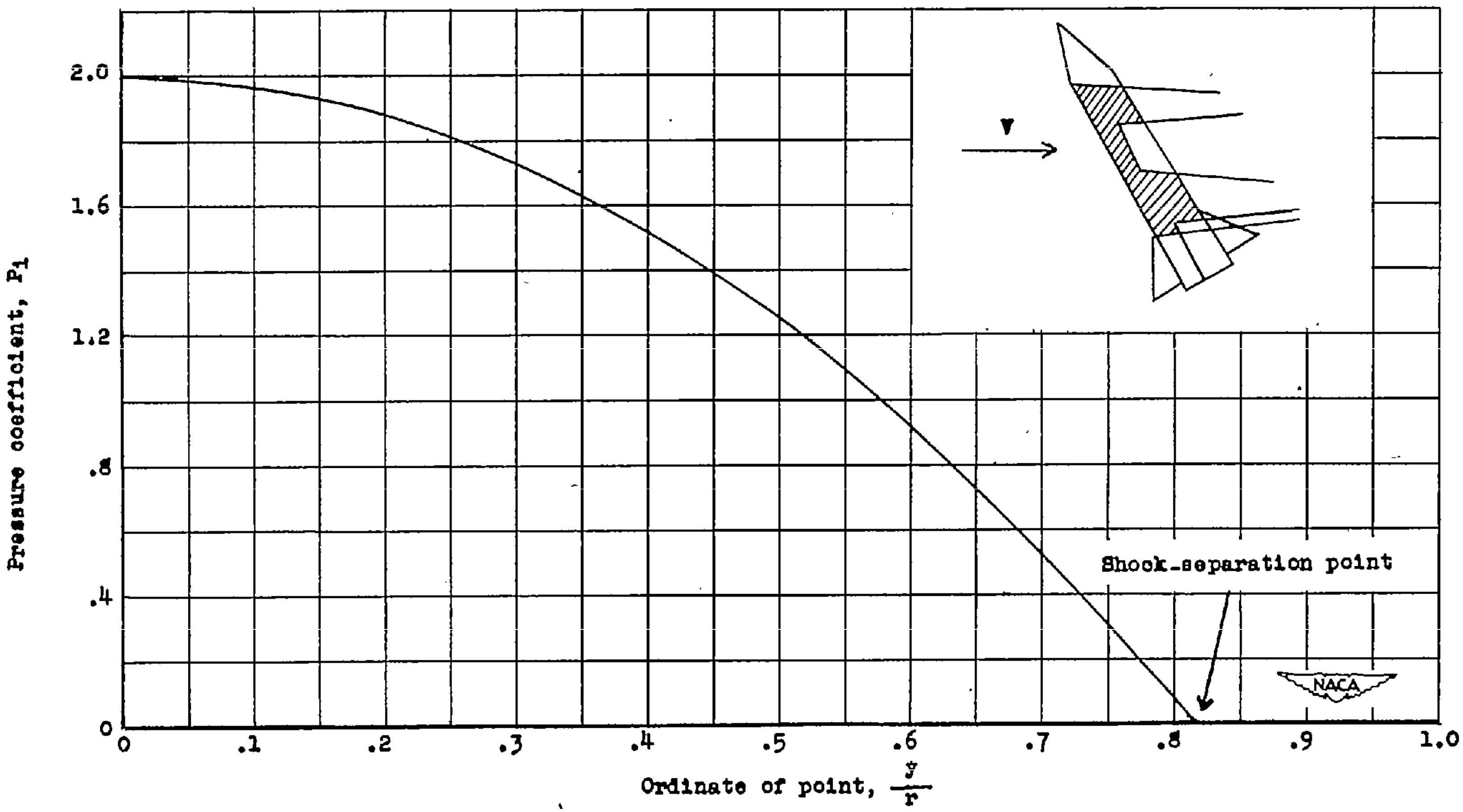

Figure 5.- Pressure distribution over section of cylinder investigated.

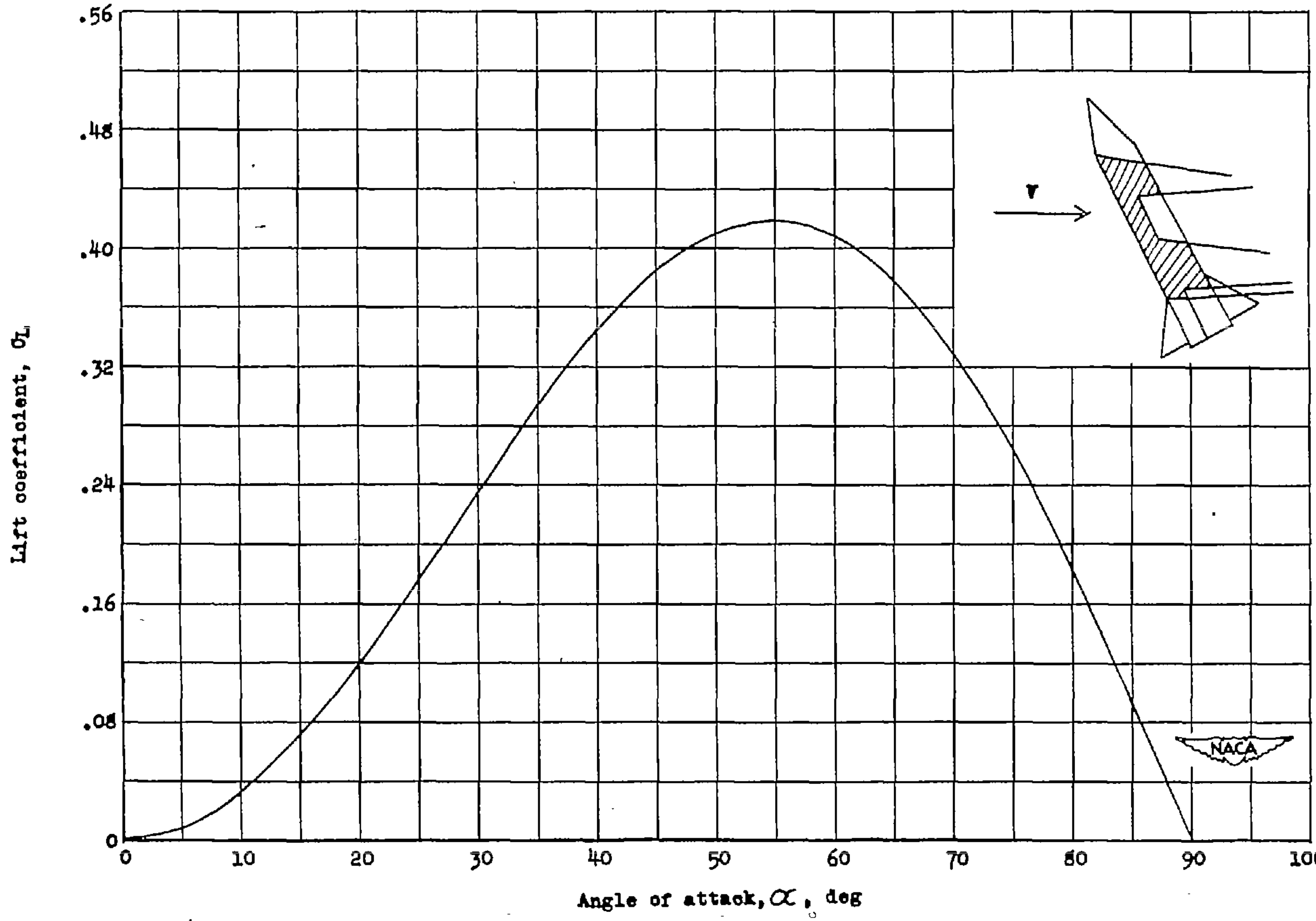

Figure 6.- Lift coefficient for section of cylinder investigated.

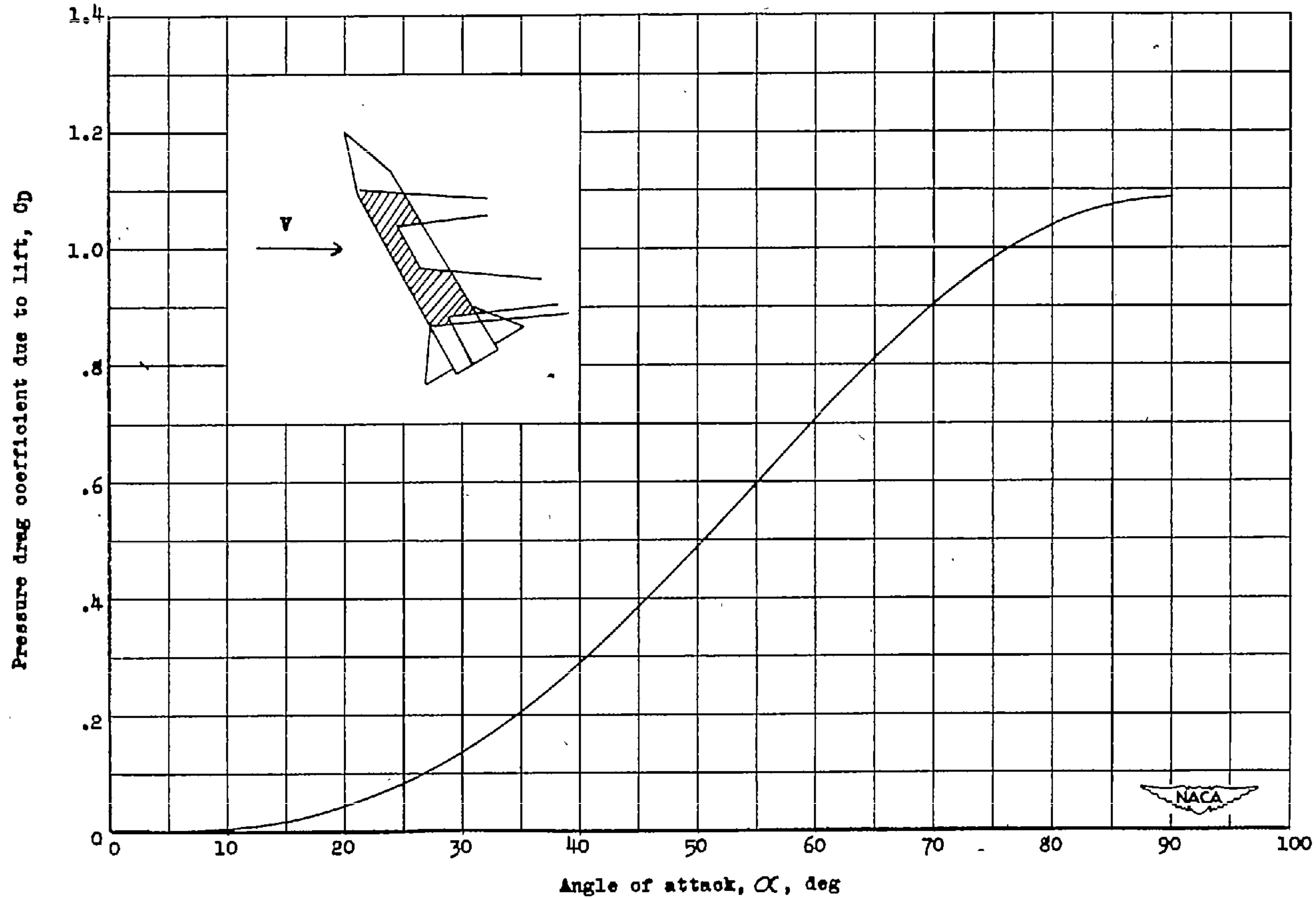

Figure 7.- Pressure drag coefficient for section of cylinder investigated.

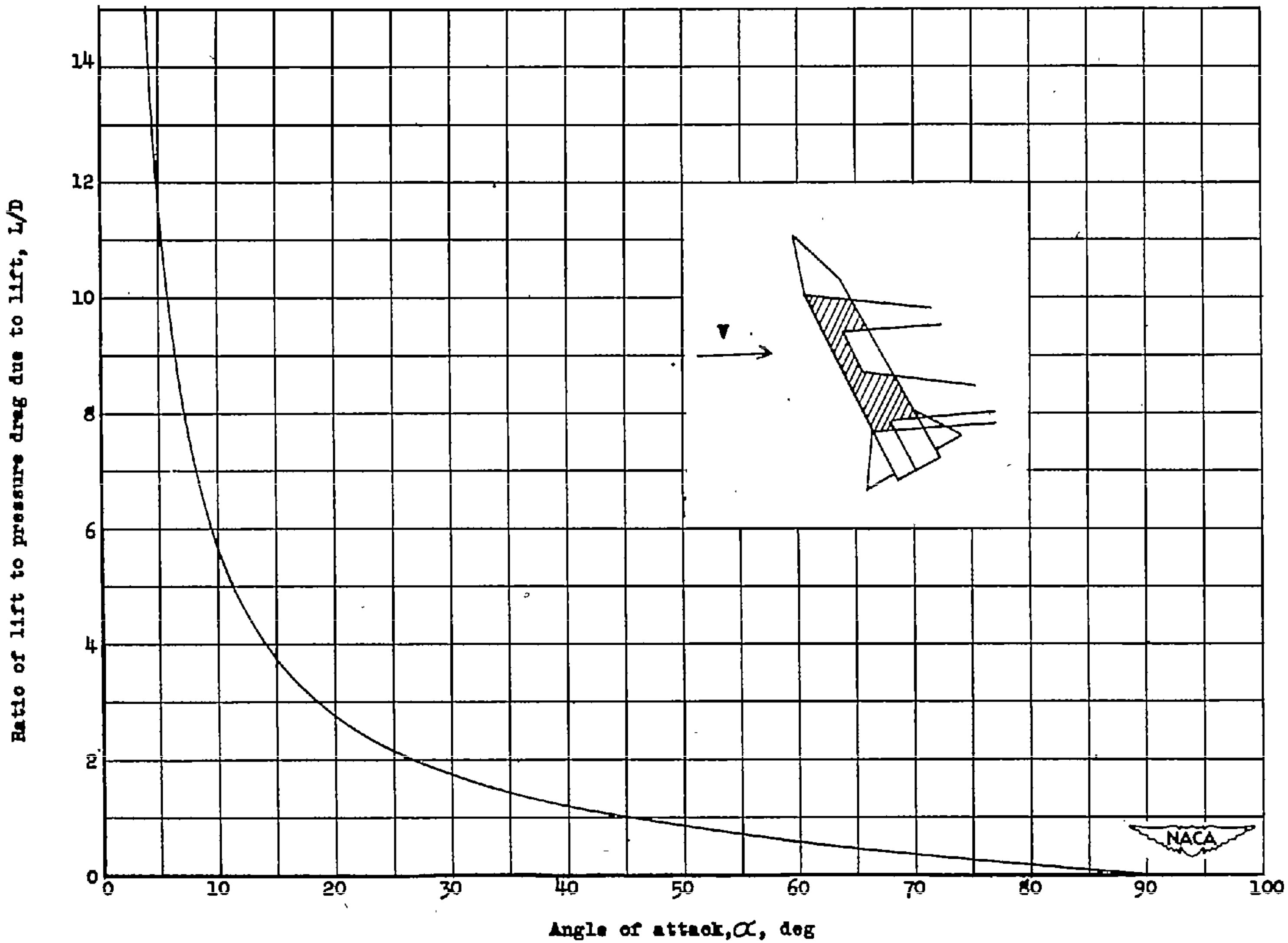

Figure 6.- Ratio of lift to pressure drag due to lift for section of cylinder investigated.

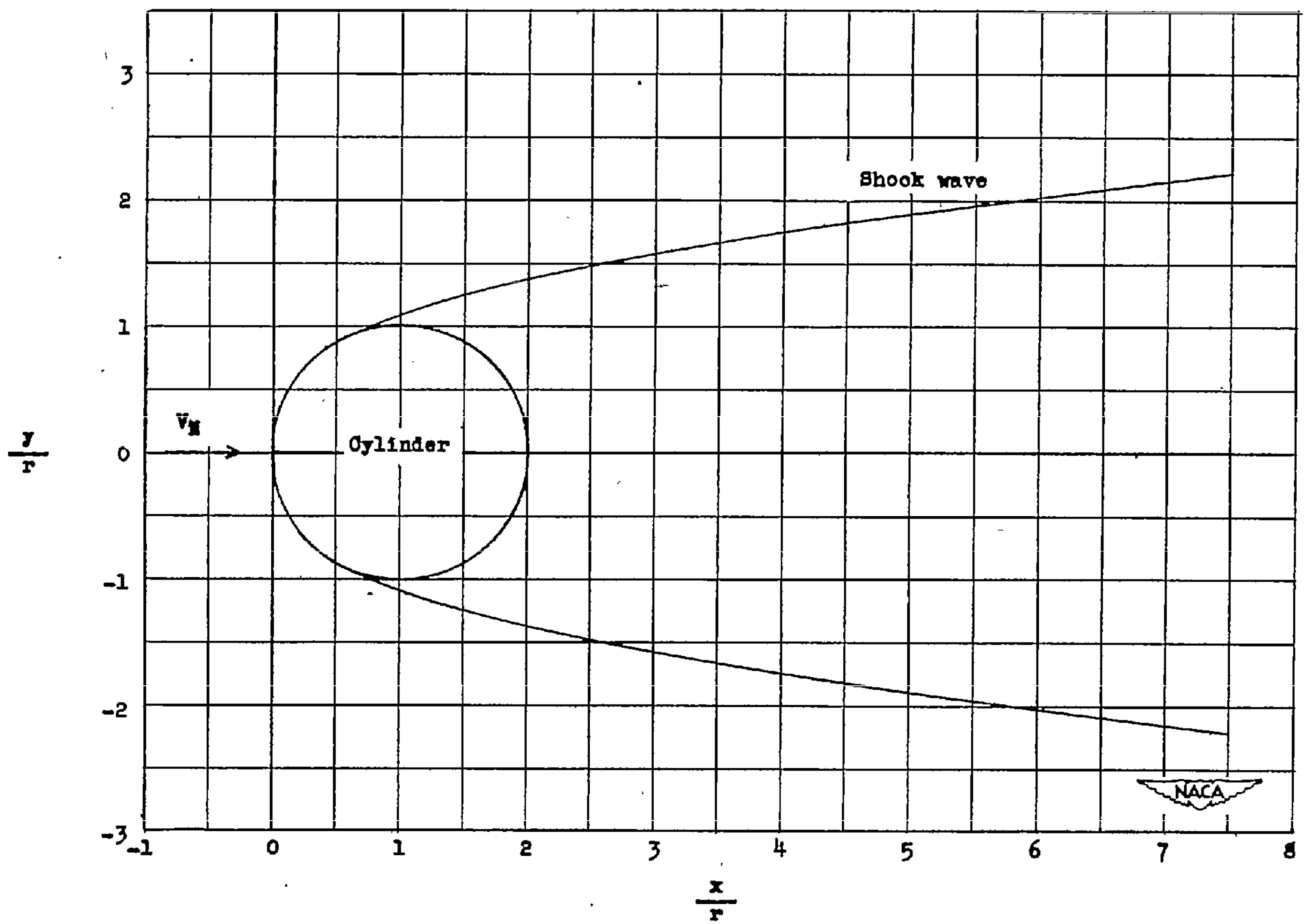

Figure 9.- Calculated shock-wave shape.

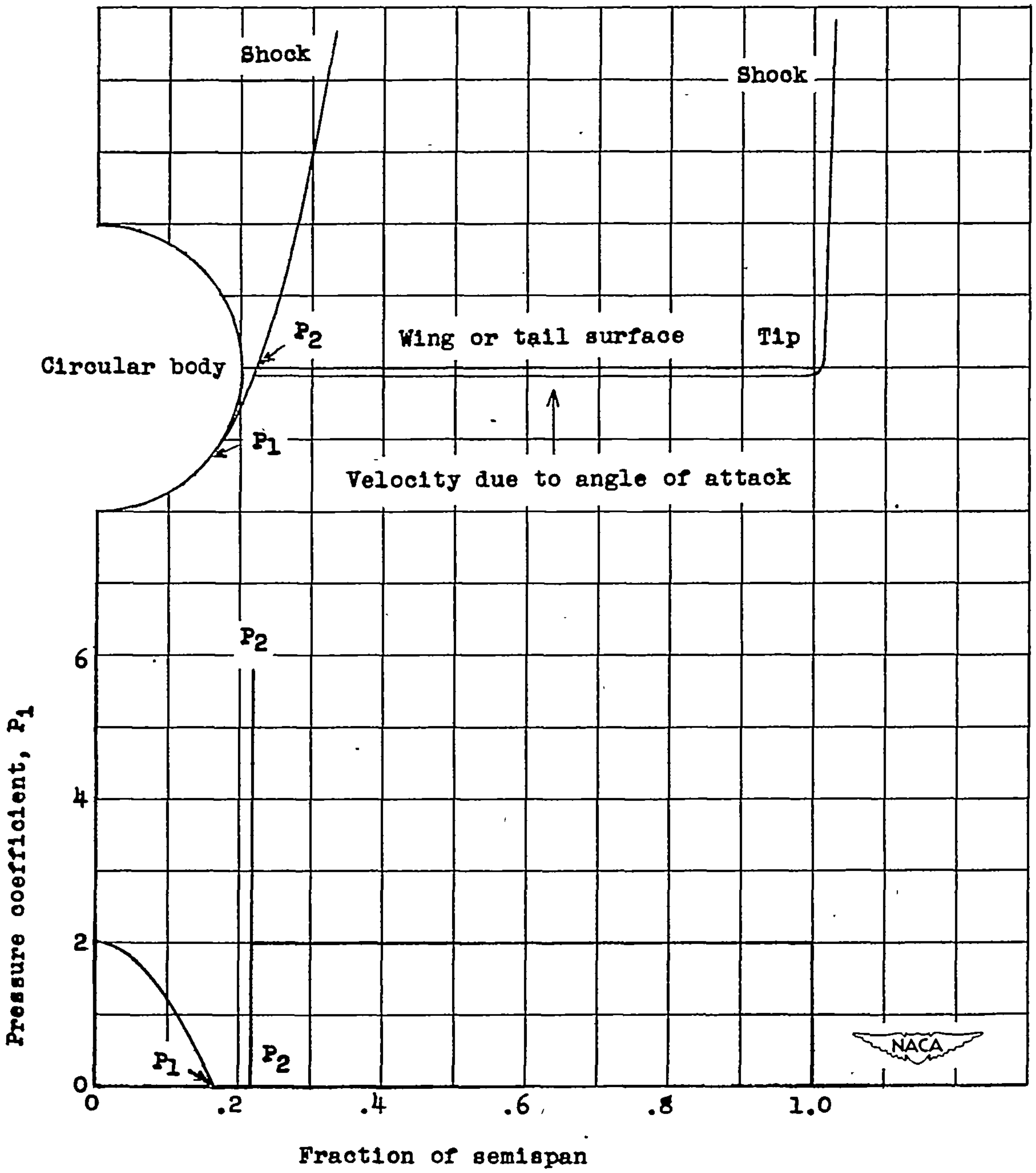

Figure 10.- Spanwise load distribution for wing-body combination.